Geometric Return

It is easiest way to explain the geometric return is by comparing it with the arithmetic return.

The arithmetic average return is sum of the returns divided by the number of returns. If the number of returns is n, then the equation for calculating the arithmetic average return is as follows:

$$r_{arithmetic} = \frac{(r_1 + r_2 + \cdots r_n)}{n}$$

The geometric average return is a compound return, and it is the product of the returns taken to the nth root.

$$r_{geometric} = \sqrt[n]{[(1 + r_1) * (1 + r_2) \ldots * (1 + r_n)]} - 1$$

The geometric return is much more accurate than an arithmetic return, and you should use it whenever possible. A simple example will illustrate this fact.

If a stock goes up by 100 percent one year and then drops by 50 percent the next year, it will wind up back where it started. Its return over those two years will be 0.

However, the arithmetic return will actually show that the stock had an average return of 25 percent.

$$r_{arithmetic} = \frac{(1.00 - 0.50)}{2} = 0.25$$

On the other hand, the geometric average will show that the stock had an average return of 0, and ended back where it started.

$$r_{geometric} = \sqrt[2]{(2.00)(0.50)} - 1 = 0.0$$

One interesting thing to note about the geometric return is that the geometric return is approximately equal to the arithmetic return minus one-half the standard deviation squared.

$$r_{geometric} \approx r_{arithmetic} - 1/2\sigma^2$$